SOLUTION

1. Let $x+1$ be the number of students in the class.

Hence, the total score of all students in the class is $75x+96$.

Similarly, the total score can also be expressed as $78x+42$.

Hence, $78x+42=75x+96$, which implies that $x=18$.

Therefore, there are 19 students in the class.

2. Observe that if $m \geq 10$, then $m!$ is divisible by 100. Hence, the last two digits of $1!+4!+7!+10!+\cdots+2023!$ is the same as the last two digits of $1!+4!+7!$.

Since $1!=1$, $4!=24$ and $7!=5040$, the last two digits is $1+24+40=65$.

3. Simplify the multiplication as follows:

$$9900\times\left(1-\frac{4}{3^2}\right)\times\left(1-\frac{4}{4^2}\right)\times\left(1-\frac{4}{5^2}\right)\times\cdots\times\left(1-\frac{4}{100^2}\right)$$

$$=9900\times\left(\frac{3^2-2^2}{3^2}\right)\times\left(\frac{4^2-2^2}{4^2}\right)\times\left(\frac{5^2-2^2}{5^2}\right)\times\cdots\times\left(\frac{100^2-2^2}{100^2}\right)$$

$$=9900\times\left(\frac{5\times1}{3^2}\right)\times\left(\frac{6\times2}{4^2}\right)\times\left(\frac{7\times3}{5^2}\right)\times\cdots\times\left(\frac{102\times98}{100^2}\right)$$

$$=9900\times\frac{1\times2\times101\times102}{3\times4\times99\times100}$$

$$=\frac{2\times101\times102}{3\times4}$$

$$=1717$$

4. Since every row and column has grey squares, thus at least 4 moves are required to get a grid of all white squares. We shall show that only 4 moves are required to achieve this. One such order of moves is Row 1, Row 4, Column 2, Column 3.

5. Among the whole numbers from 1 to 200, there are 100 even numbers, so 2^{100} is a factor of the product. There are only 40 numbers that are divisible by 5, among which eight of the numbers are also divisible by $5^2 = 25$. Moreover, the number 125 is also divisible by $5^3 = 125$. Hence, if we decompose the product into a product of prime factors, the prime factor 5 appears exactly 49 times, whereas the prime factor 2 appears more than 49 times. Since $10 = 2 \times 5$, we may write the product as $M \times 10^{49}$ for some whole number *M* that does not end with 0. Hence, the product ends with 49 zeros.

6. Let $[PQR]$ denote the area of triangle *PQR* in cm^2.

Let $[EFG] = x$. Since $AE = EB$, we have $[EBF] = [AEF] = 3 + x$.

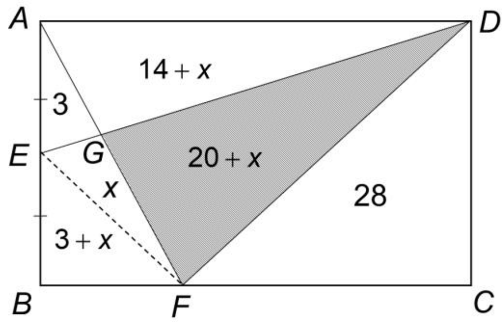
Next, observe that $[ABF] + [DFC] = 34 + 2x = \frac{1}{2} \times (\text{Area of rectangle } ABCD)$.

Hence, $[AED] = \frac{1}{4} \times (\text{Area of rectangle } ABCD) = 17 + x$, and

$[AGD] = 17 + x - [AEG] = 14 + x$.

Similarly, $[ADF] = \frac{1}{2} \times (\text{Area of rectangle } ABCD) = 34 + 2x$, and

$[DFG] = [ADF] - [ADG] = 20 + x$.



Furthermore, we have $\frac{14 + x}{3} = \frac{DG}{EG} = \frac{20 + x}{x}$.

Hence, $x(14 + x) = 3(20 + x)$, which implies that $x^2 + 11x = 60$.

By considering positive whole numbers *x* starting from 1, 2, 3, etc., we observe that only $x = 4$ will satisfy the above equation.

In conclusion, $[DFG] = 24$.

7. Suppose the number of trees that Alicia and Benjamin had planted is *m* and *n* respectively.

From the conditions, we know that $\frac{4n}{7}=\frac{7m}{11}$, which means that $44n=49m$.

So the minimum value of *n* is 49 while the minimum value of *m* is 44. So the minimum number of trees they have planted when both of them have completed their tasks is $m+n=49+44=93$.

8. We have $\left(1000a+\overline{bcd}\right)-\left(10\overline{bcd}+a\right)=2007$.

Simplifying gives $999a-9\overline{bcd}=2007$, which implies that $111a-\overline{bcd}=223$.
Note that the digit *a* has to be at least 3 in order for the left-hand side to yield a positive number. Hence, *a* can be 3, 4, 5, 6, 7, 8 or 9.
For each *a* mentioned above, the number $111a-223$ is a whole number less than 1000.

Since $111a-223=\overline{bcd}$, each *a* will give rise to exactly one corresponding value for each of the digits *b*, *c* and *d*.

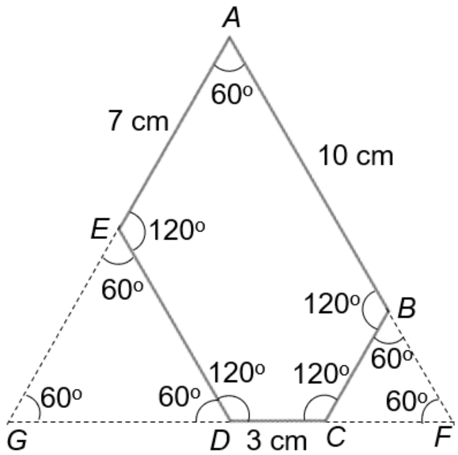
In conclusion, there are 7 such four-digit numbers $\overline{abcd}$.

9. We note that a one-digit number cannot be a lucky number since we cannot divide the digits into two groups.
For two-digit numbers, the digits have to be the same in order to be lucky. However, only 11 is a prime number. The rest of the 2-digit lucky numbers are also divisible by 11 and are hence not prime.

Next, we consider numbers of the form $\overline{1bc}$.
If $1=b+c$, then either $b=0, c=1$ or $b=1, c=0$. However, only 101 is a prime number; 110 is not prime.
If $1+b=c$, since prime numbers larger than 2 can only end with the digits 1, 3, 7 or 9, by considering these possible values as values of *c*, only 101 and 167 are prime and lucky.

If $1+c=b$, we will not have any lucky numbers that are also prime, since such numbers would be divisible by 11 as well.
In conclusion, the required sum is $11+101+167=279$.

10. Since the sum of interior angles of a pentagon is $(5-2)\times180^\circ=540^\circ$, we have $\angle BAE=540^\circ-4(120^\circ)=60^\circ$. Now, we construct two equilateral triangles *BCF* and *DEG* as shown below.



Observe that $\triangle AGF$ is also an equilateral triangle, so $AE+EG=AB+BF=GD+DC+CF$. Since $EG=GD$ and $BF=CF$, we have $AE=DC+CF$ and $AB=GD+DC$. Adding the two equations obtained gives $AE+AB=2DC+CF+GD$. Moreover, since $ED=GD$ and $CF=BC$, we have $AE+AB=2DC+BC+ED$.

Hence, perimeter of pentagon (in cm)

$$\begin{aligned}
&=AE+AB+DC+(BC+ED)\\
&=AE+AB+DC+(AE+AB-2DC)\\
&=2AE+2AB-DC\\
&=2(7)+2(10)-3\\
&=31
\end{aligned}$$

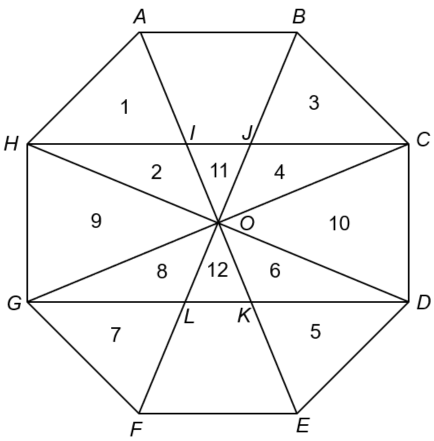
11. Let *a* be the number of men who wear glasses.
Let *b* be the number of men who do not wear glasses.
Let *c* be the number of women who wear glasses.
Let *d* be the number of women who do not wear glasses.

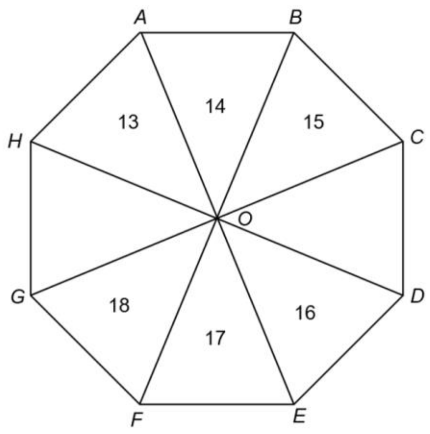
Note that $a:c=3:2$, $c:d=7:13$, so $a:c:d=21:14:26$.

Hence, if we let 40 units represent the number of women (which make up 30% of the group), then the number of units representing men (which make up 70% of the group) will be $\frac{7}{3}\times 40=\frac{280}{3}$. This implies that

$a:b:c:d=21:\left(\frac{280}{3}-21\right):14:26$. Since *a*, *b*, *c* and *d* must be whole numbers, we express the above ratio in its simplest form, which is $63:217:42:78$. We conclude that the minimum value of *a*, *b*, *c* and *d* must be 63, 217, 42 and 78 respectively. The minimum number of people in the group will be $63+217+42+78=400$.

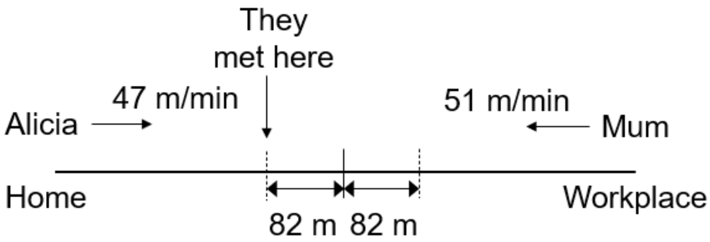
12. First, we label all vertices as shown below:





Besides the 18 triangles labelled above, there are also 10 other triangles, namely: *HJO*, *CIO*, *DLO*, *GKO*, *CGH*, *CDH*, *CDG*, *DGH*, *CHO* and *DGO*. Hence, there are 28 triangles in total.

13. Since Alicia's mum was walking at a faster pace than Alicia, they would have met nearer to their house than the workplace. Hence, Alicia's mum would have walked $82 \times 2 = 164$ metres more than Alicia when they met.



Since Alicia was 4 m/min slower than her mum, the time taken for both to meet was $164 \div 4 = 41$ minutes. Hence, the distance between the workplace from their home is $(47+51) \times 41 = 4018$ metres.

14. Let x be the amount of water (in ml) at the end. We can summarise the given information in the table below.

	Amount of Water (in ml)	Amount of Oil (in ml)
Before	$x+50$	$4x-100$
After	x	$4x$

We also know that $\frac{x+50}{4x-100} = \frac{28}{72} = \frac{7}{18}$.

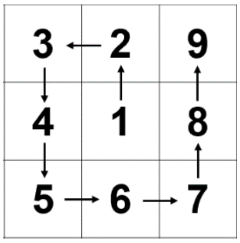
Multiplying by 4 on both sides gives $\frac{x+50}{x-25} = \frac{14}{9}$.

Hence, $14(x-25) = 9(x+50)$, which implies that $14x - 350 = 9x + 450$.

Thus, $5x = 800$, which implies that $x = 160$.

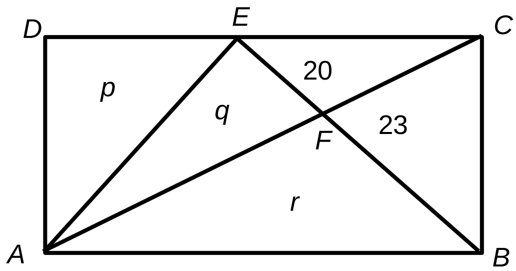
In conclusion, the initial amount of water (in ml) is $160 + 50 = 210$.

15. Observe that if n is an odd number, then n^2 is always the number on the tile that is at the top right corner of a n by n grid of tiles. For instance, 9 (which is 3^2) is positioned at the tile which is at the top right corner of the 3 by 3 grid.



Since 2025 is 45^2, 2025 is at the tile which is at the top right corner of a 45 by 45 grid of tiles. The number southwest of 2025 is $43^2 = 1849$. Since 2023th tile is 2 tiles below the 2025th tile, the number southwest of 2023 is also 2 tiles below the 1849th tile. Hence, the required number is 1847.

16. Join AE.



Let the numbers shown within the polygon represent its area (in cm^2).
Note that AEB is half of the rectangle $ABCD$; ACD is also half of rectangle $ABCD$. Hence, we have $q+r=p+20+23$ and $p+q+20=r+23$.

Take the sum of these two equations,

$(q+r)+(p+q+20)=(p+20+23)+(r+23)$ gives $q=23$.

Also, the triangles EFC and BFC are of the same height, so $\frac{EF}{FB}=\frac{20}{23}$.

Now, the triangles EFA and BFA are of the same height, so $\frac{EF}{FB}=\frac{q}{r}=\frac{20}{23}$.

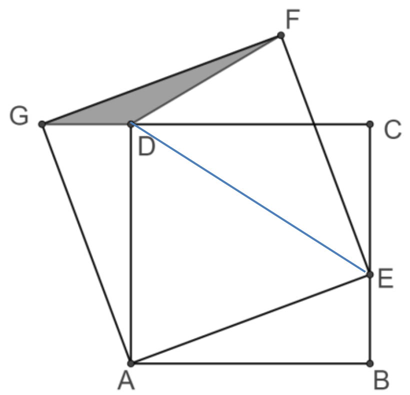
So, $r=\frac{23q}{20}=\frac{23\times 23}{20}$. Now from $p+q+20=r+23$, we have

$$p=r-20=\frac{23\times 23}{20}-20=\frac{23\times 23-20\times 20}{20}=\frac{43\times 3}{20}=\frac{129}{20}.$$

Now, $x=p+q=\frac{129}{20}+23=\frac{129+20\times 23}{20}=\frac{129+460}{20}=\frac{589}{20}$, so $20x=589$.

17. Let a be the first factor. Then the largest factor is 105a and $N = 105a^2$. As $105 = 3\times 5\times 7$. Thus, the smallest factor must be less than or equal to 3. Hence there are 2 possible values of N, i.e., $N = 105(2)^2 = 420$ and $N = 105(3)^2 = 945$. The sum is $420+945=1365$.

18.



Join *DE*, as shown. Then the area of triangle *ADE* is half of the area of the square *ABCD*. So, the area of triangle *ADE* is $\frac{1777}{2}$ cm^2. Note that the sum of area of triangles *DGF* and *ADE* is half of the area of square *AEFG*. Then the area of *DFG* is $\frac{2023}{2}-\frac{1777}{2}=\frac{246}{2}=123$ cm^2.

19. As the numbers are from 1 to 16, each row must add to ¼ of the sum of the numbers from 1 to 16 which means that *N* is 34. Once we let the left bottom corner number be A, we may fill in the rest of the blanks using the above criterion. One possible order of filling the magic square could be: A, 21-A, 20-A, 9, A-8, 29-A, A+1, 3.

1	29-A		
12	A-8		
21-A	9	A+1	3
A	4	20-A	10

Now look at the diagonal consists of the numbers 1, A-8, A+1 and 10. The sum is 34, hence $1+(A-8)+(A+1)+10=34 \Rightarrow A=15$.

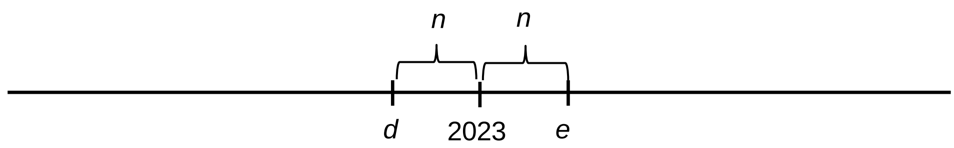
1	14		
12	7		
6	9	16	3
15	4	5	10

Now rest of the numbers can be filled in accordingly, as shown.

1	14	11	8
12	7	2	13
6	9	16	3
15	4	5	10

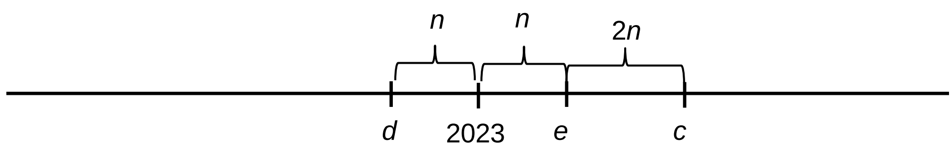
So, the three numbers in the shaded squares are 6, 7 and 11. Thus the product is $6\times7\times11=462$

20. Using the condition, we know that $\frac{d+e}{2}=2023$. We draw the number line, as shown:

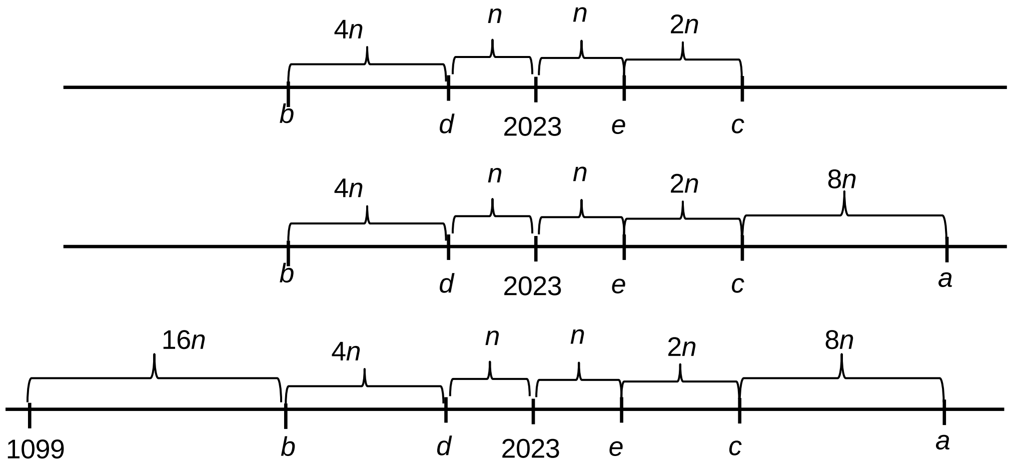


Suppose the difference between 2023 and *e* is *n*. Then the difference between *d* and 2023 is *n* too. This means that 2023 is the mid-point of *d* and *e*.

From the sequence again, we know that *e* is the mid-point of *d* and *c*. So we obtain the following:



Using the similar argument, we will obtain the following, at each stage:



From the above number line, we have $21n=2023-1099=924$ which means that $n=\frac{924}{21}=44$. Now $c=2023+3n=2023+3\times 44=2023+132=2155$.

For completeness, the sequence of $1099, a, b, c, d, e, 2023$ is

1099, 2507, 1803, **2155**, 1979, 2067, 2023.

- THE END -